

# Package ‘clugenr’

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**Title** Multidimensional Cluster Generation Using Support Lines

**Version** 1.0.4

**Description** An implementation of the clugen algorithm for generating multidimensional clusters with arbitrary distributions. Each cluster is supported by a line segment, the position, orientation and length of which guide where the respective points are placed. This package is described in Fachada & de Andrade (2023) <[doi:10.1016/j.knosys.2023.110836](https://doi.org/10.1016/j.knosys.2023.110836)>.

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**RdMacros** mathjaxr

**License** MIT + file LICENSE

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<https://github.com/clugen/clugenr>

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angle_btw	<i>Angle between two n-dimensional vectors.</i>
-----------	---

---

### Description

Typically, the angle between two vectors  $v_1$  and  $v_2$  can be obtained with:

```
acos((v1 %*% v2) / (norm(v1, "2") * norm(v2, "2")))
```

However, this approach is numerically unstable. The version provided here is numerically stable and based on the [Angle Between Vectors](#) Julia package by Jeffrey Sarnoff (MIT license), implementing an algorithm provided by Prof. W. Kahan in [these notes](#) (see page 15).

### Usage

```
angle_btw(v1, v2)
```

### Arguments

$v_1$	First vector.
$v_2$	Second vector.

### Value

Angle between  $v_1$  and  $v_2$  in radians.

### Examples

```
angle_btw(c(1.0, 1.0, 1.0, 1.0), c(1.0, 0.0, 0.0, 0.0)) * 180 / pi
```

---

angle_deltas	<i>Get angles between average cluster direction and cluster-supporting lines</i>
--------------	--

---

### Description

Determine the angles between the average cluster direction and the cluster-supporting lines. These angles are obtained from a wrapped normal distribution ( $\mu = 0$ ,  $\sigma = \text{angle\_disp}$ ) with support in the interval  $[-\pi/2, \pi/2]$ . Note this is different from the standard wrapped normal distribution, the support of which is given by the interval  $[-\pi, \pi]$ .

### Usage

```
angle_deltas(num_clusters, angle_disp)
```

### Arguments

num_clusters	Number of clusters.
angle_disp	Angle dispersion, in radians.

### Value

Angles between the average cluster direction and the cluster-supporting lines, given in radians in the interval  $[-\pi/2, \pi/2]$

### Note

This function is stochastic. For reproducibility set a PRNG seed with [set.seed](#).

### Examples

```
set.seed(123)
arad <- angle_deltas(4, pi / 8) # Angle dispersion of 22.5 degrees
arad                               # What angles deltas did we get?
arad * 180 / pi                     # Show angle deltas in degrees
```

---

clucenters	<i>Determine cluster centers using the uniform distribution</i>
------------	---

---

**Description**

Determine cluster centers using the uniform distribution, taking into account the number of clusters (`num_clusters`) and the average cluster separation (`clu_sep`).

More specifically, let  $c = \text{num\_clusters}$ ,  $s = \text{clu\_sep}$ ,  $\mathbf{o} = \text{clu\_offset}$ ,  $n = \text{length}(\text{clu\_sep})$  (i.e., number of dimensions). Cluster centers are obtained according to the following equation:

$$\mathbf{C} = c\mathbf{U} \cdot \text{diag}(\mathbf{s}) + \mathbf{1} \mathbf{o}^T$$

where  $\mathbf{C}$  is the  $c \times n$  matrix of cluster centers,  $\mathbf{U}$  is an  $c \times n$  matrix of random values drawn from the uniform distribution between -0.5 and 0.5, and  $\mathbf{1}$  is an  $c \times 1$  vector with all entries equal to 1.

**Usage**

```
clucenters(num_clusters, clu_sep, clu_offset)
```

**Arguments**

<code>num_clusters</code>	Number of clusters.
<code>clu_sep</code>	Average cluster separation ( $n \times 1$ vector).
<code>clu_offset</code>	Cluster offsets ( $n \times 1$ vector).

**Value**

A  $c \times n$  matrix containing the cluster centers.

**Note**

This function is stochastic. For reproducibility set a PRNG seed with [set.seed](#).

**Examples**

```
set.seed(321)
clucenters(3, c(30, 10), c(-50,50))
```

---

clugen

*Generate multidimensional clusters*

---

**Description**

This is the main function of **clugenr**, and possibly the only function most users will need.

**Usage**

```

clugen(
  num_dims,
  num_clusters,
  num_points,
  direction,
  angle_disp,
  cluster_sep,
  llength,
  llength_disp,
  lateral_disp,
  allow_empty = FALSE,
  cluster_offset = NA,
  proj_dist_fn = "norm",
  point_dist_fn = "n-1",
  clusizes_fn = clusizes,
  clucenters_fn = clucenters,
  llengths_fn = llengths,
  angle_deltas_fn = angle_deltas,
  seed = NA
)

```

**Arguments**

num_dims	Number of dimensions.
num_clusters	Number of clusters to generate.
num_points	Total number of points to generate.
direction	Average direction of the cluster-supporting lines. Can be a vector of length num_dims (same direction for all clusters) or a matrix of size num_clusters x num_dims (one direction per cluster).
angle_disp	Angle dispersion of cluster-supporting lines (radians).
cluster_sep	Average cluster separation in each dimension (vector of length num_dims).
llength	Average length of cluster-supporting lines.
llength_disp	Length dispersion of cluster-supporting lines.
lateral_disp	Cluster lateral dispersion, i.e., dispersion of points from their projection on the cluster-supporting line.
allow_empty	Allow empty clusters? FALSE by default.
cluster_offset	Offset to add to all cluster centers (vector of length num_dims). By default there will be no offset.
proj_dist_fn	Distribution of point projections along cluster-supporting lines, with three possible values: <ul style="list-style-type: none"> <li>"norm" (default): Distribute point projections along lines using a normal distribution (<math>\mu = \text{line\_center}</math>, <math>\sigma = \text{llength}/6</math>).</li> <li>"unif": Distribute points uniformly along the line.</li> </ul>

- User-defined function, which accepts two parameters, line length (double) and number of points (integer), and returns a vector containing the distance of each point projection to the center of the line. For example, the "norm" option roughly corresponds to `function(1, n) stats::rnorm(n, sd = 1 / 6)`.
- `point_dist_fn` Controls how the final points are created from their projections on the cluster-supporting lines, with three possible values:
- "n-1" (default): Final points are placed on a hyperplane orthogonal to the cluster-supporting line, centered at each point's projection, using the normal distribution ( $\mu = 0$ ,  $\sigma = \text{lateral\_disp}$ ). This is done by the `clupoints_n_1` function.
  - "n": Final points are placed around their projection on the cluster-supporting line using the normal distribution ( $\mu = 0$ ,  $\sigma = \text{lateral\_disp}$ ). This is done by the `clupoints_n` function.
  - User-defined function: The user can specify a custom point placement strategy by passing a function with the same signature as `clupoints_n_1` and `clupoints_n`.
- `clusizes_fn` Distribution of cluster sizes. By default, cluster sizes are determined by the `clusizes` function, which uses the normal distribution ( $\mu = \text{num\_points}/\text{num\_clusters}$ ,  $\sigma = \mu/3$ ), and assures that the final cluster sizes add up to `num_points`. This parameter allows the user to specify a custom function for this purpose, which must follow `clusizes` signature. Note that custom functions are not required to strictly obey the `num_points` parameter. Alternatively, the user can specify a vector of cluster sizes directly.
- `clucenters_fn` Distribution of cluster centers. By default, cluster centers are determined by the `clucenters` function, which uses the uniform distribution, and takes into account the `num_clusters` and `cluster_sep` parameters for generating well-distributed cluster centers. This parameter allows the user to specify a custom function for this purpose, which must follow `clucenters` signature. Alternatively, the user can specify a matrix of size `num_clusters` x `num_dims` with the exact cluster centers.
- `llengths_fn` Distribution of line lengths. By default, the lengths of cluster-supporting lines are determined by the `llengths` function, which uses the folded normal distribution ( $\mu = \text{llength}$ ,  $\sigma = \text{llength\_disp}$ ). This parameter allows the user to specify a custom function for this purpose, which must follow `llengths` signature. Alternatively, the user can specify a vector of line lengths directly.
- `angle_deltas_fn` Distribution of line angle differences with respect to direction. By default, the angles between the main direction of each cluster and the final directions of their cluster-supporting lines are determined by the `angle_deltas` function, which uses the wrapped normal distribution ( $\mu = 0$ ,  $\sigma = \text{angle\_disp}$ ) with support in the interval  $[-\pi/2, \pi/2]$ . This parameter allows the user to specify a custom function for this purpose, which must follow `angle_deltas` signature. Alternatively, the user can specify a vector of angle deltas directly.
- `seed` An integer used to initialize the PRNG, allowing for reproducible results. If specified, `seed` is simply passed to `set.seed`.

**Details**

If a custom function was given in the `clusizes_fn` parameter, it is possible that `num_points` may have a different value than what was specified in the `num_points` parameter.

The terms "average" and "dispersion" refer to measures of central tendency and statistical dispersion, respectively. Their exact meaning depends on the optional arguments.

**Value**

A named list with the following elements:

- `points`: A `num_points` x `num_dims` matrix with the generated points for all clusters.
- `clusters`: A `num_points` factor vector indicating which cluster each point in `points` belongs to.
- `projections`: A `num_points` x `num_dims` matrix with the point projections on the cluster-supporting lines.
- `sizes`: A `num_clusters` x 1 vector with the number of points in each cluster.
- `centers`: A `num_clusters` x `num_dims` matrix with the coordinates of the cluster centers.
- `directions`: A `num_clusters` x `num_dims` matrix with the final direction of each cluster-supporting line.
- `angles`: A `num_clusters` x 1 vector with the angles between the cluster-supporting lines and the main direction.
- `lengths`: A `num_clusters` x 1 vector with the lengths of the cluster-supporting lines.

**Note**

This function is stochastic. For reproducibility set a PRNG seed with [set.seed](#).

**Examples**

```
# 2D example
x <- clugen(2, 5, 1000, c(1, 3), 0.5, c(10, 10), 8, 1.5, 2)
graphics::plot(x$points, col = x$clusters, xlab = "x", ylab = "y", asp = 1)
# 3D example
x <- clugen(3, 5, 1000, c(2, 3, 4), 0.5, c(15, 13, 14), 7, 1, 2)
```

---

clumerge

*Merges the fields (specified in fields) of two or more data sets*


---

**Description**

Merges the fields (specified in `fields`) of two or more data sets (passed as lists). The fields to be merged need to have the same number of columns. The corresponding merged field will contain the rows of the fields to be merged, and will have a common "supertype".

**Usage**

```
clumerge(..., fields = c("points", "clusters"), clusters_field = "clusters")
```

**Arguments**

`...` One or more cluster data sets (in the form of lists) whose fields are to be merged.

`fields` Fields to be merged, which must exist in the data sets given in `...`

`clusters_field` Field containing the integer cluster labels. If specified, cluster assignments in individual datasets will be updated in the merged dataset so that clusters are considered separate.

**Details**

The `clusters_field` parameter specifies a field containing integers that identify the cluster to which the respective points belongs to. If `clusters_field` is specified (by default it's specified as "clusters"), cluster assignments in individual datasets will be updated in the merged dataset so that clusters are considered separate. This parameter can be set to NA, in which case no field will be considered as a special cluster assignments field.

This function can be used to merge data sets generated with the [clugen](#) function, by default merging the `points` and `clusters` fields in those data sets. It also works with arbitrary data by specifying alternative fields in the `fields` parameter. It can be used, for example, to merge third-party data with [clugen](#)-generated data.

**Value**

A list whose fields consist of the merged fields in the original data sets.

**Examples**

```
a <- clugen(2, 5, 100, c(1, 3), 0.5, c(10, 10), 8, 1.5, 2)
b <- clugen(2, 3, 250, c(-1, 3), 0.5, c(13, 14), 7, 1, 2)
ab <- clumerge(a, b)
```

---

clupoints\_n

*Create points from their projections on a cluster-supporting line*

---

**Description**

Each point is placed around its projection using the normal distribution ( $\mu = 0, \sigma = \text{lat\_disp}$ ).

**Usage**

```
clupoints_n(projs, lat_disp, line_len, clu_dir, clu_ctr)
```

**Arguments**

projs	Point projections on the cluster-supporting line ( $p \times n$ matrix).
lat_disp	Standard deviation for the normal distribution, i.e., cluster lateral dispersion.
line_len	Length of cluster-supporting line (ignored).
clu_dir	Direction of the cluster-supporting line.
clu_ctr	Center position of the cluster-supporting line (ignored).

**Details**

This function's main intended use is by the main [clugen](#) function, generating the final points when the `point_dist_fn` parameter is set to "n".

**Value**

Generated points ( $p \times n$  matrix).

**Note**

This function is stochastic. For reproducibility set a PRNG seed with [set.seed](#).

**Examples**

```
set.seed(123)
ctr <- c(0, 0)
dir <- c(1, 0)
pdist <- c(-0.5, -0.2, 0.1, 0.3)
proj <- points_on_line(ctr, dir, pdist)
clupoints_n(proj, 0.01, NA, dir, NA)
```

---

clupoints\_n\_1

*Create points from their projections on a cluster-supporting line*

---

**Description**

Each point is placed on a hyperplane orthogonal to that line and centered at the point's projection, using the normal distribution ( $\mu = 0, \sigma = \text{lat\_disp}$ ).

**Usage**

```
clupoints_n_1(projs, lat_disp, line_len, clu_dir, clu_ctr)
```

**Arguments**

projs	Point projections on the cluster-supporting line ( $p \times n$ matrix).
lat_disp	Standard deviation for the normal distribution, i.e., cluster lateral dispersion.
line_len	Length of cluster-supporting line (ignored).
clu_dir	Direction of the cluster-supporting line.
clu_ctr	Center position of the cluster-supporting line (ignored).

**Details**

This function's main intended use is by the main [clugen](#) function, generating the final points when the `point_dist_fn` parameter is set to "n-1".

**Value**

Generated points ( $p \times n$  matrix).

**Note**

This function is stochastic. For reproducibility set a PRNG seed with [set.seed](#).

**Examples**

```
set.seed(123)
ctr <- c(0, 0)
dir <- c(1, 0)
pdist <- c(-0.5, -0.2, 0.1, 0.3)
proj <- points_on_line(ctr, dir, pdist)
clupoints_n_1(proj, 0.1, NA, dir, NA)
```

---

clupoints\_n\_1\_template

*Create points from their projections on a cluster-supporting line*

---

**Description**

Generate points from their  $n$ -dimensional projections on a cluster-supporting line, placing each point on a hyperplane orthogonal to that line and centered at the point's projection. The function specified in `dist_fn` is used to perform the actual placement.

**Usage**

```
clupoints_n_1_template(projs, lat_disp, clu_dir, dist_fn)
```

**Arguments**

<code>projs</code>	Point projections on the cluster-supporting line ( $p \times n$ matrix).
<code>lat_disp</code>	Dispersion of points from their projection.
<code>clu_dir</code>	Direction of the cluster-supporting line (unit vector).
<code>dist_fn</code>	Function to place points on a second line, orthogonal to the first.

**Details**

This function is used internally by [clupoints\\_n\\_1](#) and may be useful for constructing user-defined final point placement strategies for the `point_dist_fn` parameter of the main [clugen](#) function.

**Value**

Generated points ( $p \times n$  matrix).

**Note**

This function is stochastic. For reproducibility set a PRNG seed with [set.seed](#).

**Examples**

```
set.seed(123)
ctr <- c(0, 0)
dir <- c(1, 0)
pdist <- c(-0.5, -0.2, 0.1, 0.3)
proj <- points_on_line(ctr, dir, pdist)
clupoints_n_1_template(proj, 0, dir, function(p, l) stats::runif(p))
```

---

clusizes

*Determine cluster sizes, i.e., the number of points in each cluster*


---

**Description**

Cluster sizes are determined using the normal distribution ( $\mu = \text{num\_points} / \text{num\_clusters}$ ,  $\sigma = \mu/3$ ), and then assuring that the final cluster sizes add up to `num_points` via the [fix\\_num\\_points](#) function.

**Usage**

```
clusizes(num_clusters, num_points, allow_empty)
```

**Arguments**

<code>num_clusters</code>	Number of clusters.
<code>num_points</code>	Total number of points.
<code>allow_empty</code>	Allow empty clusters?

**Value**

Number of points in each cluster (vector of length `num_clusters`).

**Note**

This function is stochastic. For reproducibility set a PRNG seed with [set.seed](#).

**Examples**

```
set.seed(123)
sizes <- clusizes(4, 1000, TRUE)
sizes
sum(sizes)
```

---

fix_empty	<i>Certify that, given enough points, no clusters are left empty</i>
-----------	--

---

### Description

Certifies that, given enough points, no clusters are left empty. This is done by removing a point from the largest cluster and adding it to an empty cluster while there are empty clusters. If the total number of points is smaller than the number of clusters (or if the `allow_empty` parameter is set to `TRUE`), this function does nothing.

### Usage

```
fix_empty(clu_num_points, allow_empty = FALSE)
```

### Arguments

`clu_num_points` Number of points in each cluster (vector of size  $c$ ), where  $c$  is the number of clusters.

`allow_empty` Allow empty clusters?

### Details

This function is used internally by `clusizes` and might be useful for custom cluster sizing implementations given as the `clusizes_fn` parameter of the main `clugen` function.

### Value

Number of points in each cluster, after being fixed by this function (vector of size  $c$ ).

### Examples

```
clusters <- c(3, 4, 5, 0, 0) # A vector with some empty elements
clusters <- fix_empty(clusters) # Apply this function
clusters # Check that there's no more empty elements
```

---

fix_num_points	<i>Certify that array values add up to a specific total</i>
----------------	---

---

### Description

Certifies that the values in the `clu_num_points` array, i.e. the number of points in each cluster, add up to `num_points`. If this is not the case, the `clu_num_points` array is modified in-place, incrementing the value corresponding to the smallest cluster while `sum(clu_num_points) < num_points`, or decrementing the value corresponding to the largest cluster while `sum(clu_num_points) > num_points`.

**Usage**

```
fix_num_points(clu_num_points, num_points)
```

**Arguments**

`clu_num_points` Number of points in each cluster (vector of size  $c$ ), where  $c$  is the number of clusters.

`num_points` The expected total number of points.

**Details**

This function is used internally by `clusizes` and might be useful for custom cluster sizing implementations given as the `clusizes_fn` parameter of the main `clugen` function.

**Value**

Number of points in each cluster, after being fixed by this function.

**Examples**

```
clusters <- c(1, 6, 3)           # 10 total points
clusters <- fix_num_points(clusters, 12) # But we want 12 total points
clusters                         # Check that we now have 12 points
```

---

llengths

*Determine length of cluster-supporting lines*


---

**Description**

Line lengths are determined using the folded normal distribution ( $\mu = \text{llength}$ ,  $\sigma = \text{llength\_disp}$ ).

**Usage**

```
llengths(num_clusters, llength, llength_disp)
```

**Arguments**

`num_clusters` Number of clusters.

`llength` Average line length.

`llength_disp` Line length dispersion.

**Value**

Lengths of cluster-supporting lines (vector of size `num_clusters`).

**Note**

This function is stochastic. For reproducibility set a PRNG seed with [set.seed](#).

**Examples**

```
set.seed(123)
llengths(4, 20, 3.5)
```

---

points\_on\_line

*Determine coordinates of points on a line*

---

**Description**

Determine coordinates of points on a line with `center` and `direction`, based on the distances from the center given in `dist_center`.

This works by using the vector formulation of the line equation assuming `direction` is a  $n$ -dimensional unit vector. In other words, considering  $\mathbf{d} = \text{as.matrix}(\text{direction})$  ( $n \times 1$  vector),  $\mathbf{c} = \text{as.matrix}(\text{center})$  ( $n \times 1$  vector), and  $\mathbf{w} = \text{as.matrix}(\text{dist\_center})$  ( $p \times 1$  vector), the coordinates of points on the line are given by:

$$\mathbf{P} = \mathbf{1} \mathbf{c}^T + \mathbf{w} \mathbf{d}^T$$

where  $\mathbf{P}$  is the  $p \times n$  matrix of point coordinates on the line, and  $\mathbf{1}$  is a  $p \times 1$  vector with all entries equal to 1.

**Usage**

```
points_on_line(center, direction, dist_center)
```

**Arguments**

<code>center</code>	Center of the line ( $n$ -component vector).
<code>direction</code>	Line direction ( $n$ -component unit vector).
<code>dist_center</code>	Distance of each point to the center of the line ( $n$ -component vector, where $n$ is the number of points).

**Value**

Coordinates of points on the specified line ( $p \times n$  matrix).

**Examples**

```
points_on_line(c(5, 5), c(1, 0), seq(-4, 4, length.out=5)) # 2D, 5 points
points_on_line(c(-2, 0, 0, 2), c(0, 0, -1, 0), c(10, -10)) # 4D, 2 points
```

---

rand\_ortho\_vector      *Get a random unit vector orthogonal to u.*

---

**Description**

Get a random unit vector orthogonal to u.

**Usage**

```
rand_ortho_vector(u)
```

**Arguments**

u                      A unit vector.

**Value**

A random unit vector orthogonal to u.

**Note**

This function is stochastic. For reproducibility set a PRNG seed with [set.seed](#).

**Examples**

```
r <- stats::runif(3)      # Get a random 3D vector
r <- r / norm(r, "2")    # Normalize it
o <- rand_ortho_vector(r) # Get a random unit vector orthogonal to r
r %*% o                  # Check that r and o are orthogonal (result should be ~0)
```

---

rand\_unit\_vector      *Get a random unit vector with num\_dims components.*

---

**Description**

Get a random unit vector with num\_dims components.

**Usage**

```
rand_unit_vector(num_dims)
```

**Arguments**

num\_dims              Number of components in vector (i.e. vector size).

**Value**

A random unit vector with `num_dims` components.

**Note**

This function is stochastic. For reproducibility set a PRNG seed with [set.seed](#).

**Examples**

```
r <- rand_unit_vector(4)
norm(r, "2")
```

---

`rand_vector_at_angle` *Get a random unit vector at a given angle with another vector.*

---

**Description**

Get a random unit vector which is at `angle` radians of vector `u`. Note that `u` is expected to be a unit vector itself.

**Usage**

```
rand_vector_at_angle(u, angle)
```

**Arguments**

<code>u</code>	Unit vector with $n$ components.
<code>angle</code>	Angle in radians.

**Value**

Random unit vector with  $n$  components which is at `angle` radians with vector `u`.

**Note**

This function is stochastic. For reproducibility set a PRNG seed with [set.seed](#).

**Examples**

```
u <- c(1.0, 0, 0.5, -0.5)      # Define a 4D vector
u <- u / norm(u, "2")        # Normalize the vector
v <- rand_vector_at_angle(u, pi / 4) # Get a vector at 45 degrees
arad <- acos((u %*% v) / norm(u, "2") * norm(v, "2")) # Get angle in radians
arad * 180 / pi # Convert to degrees, should be close to 45 degrees
```

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